

Resultants of Planar Forces

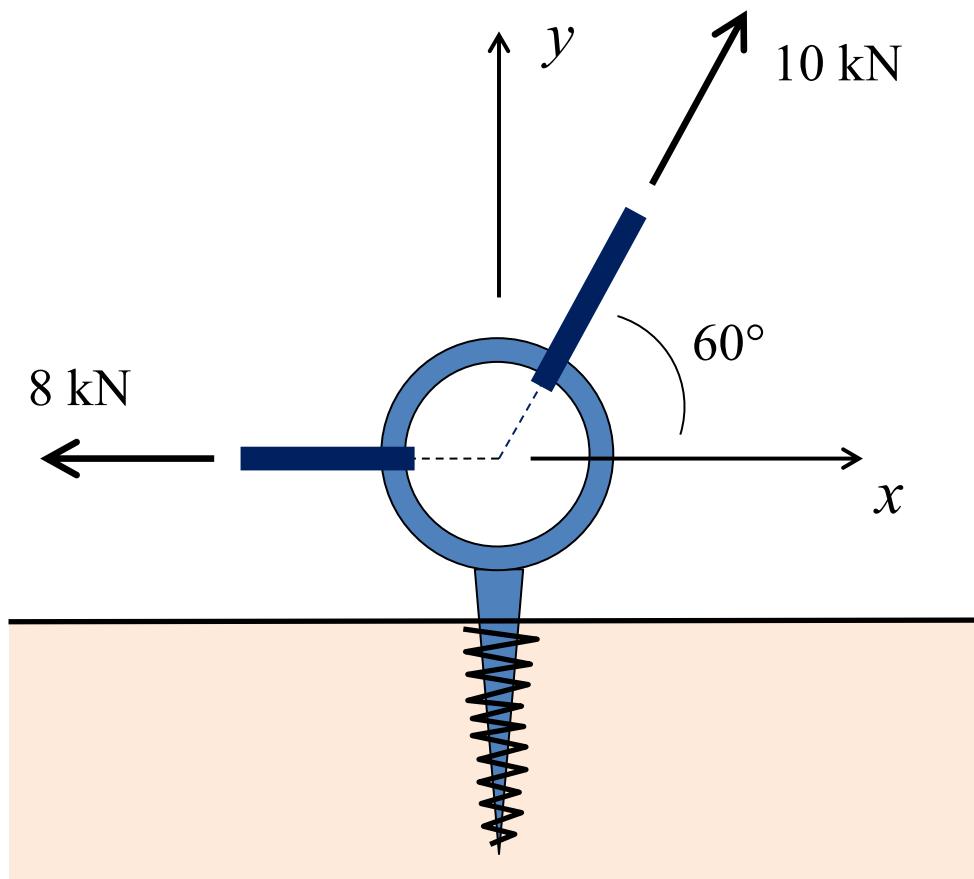
Example Problem

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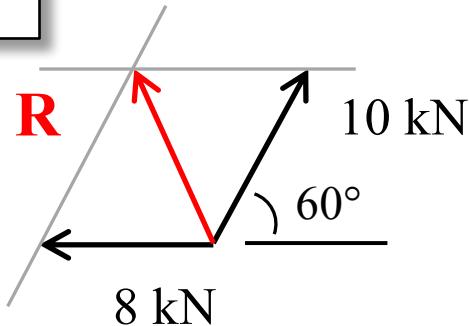
Determine the resultant of the two forces acting on the eye-bolt using:

1. The parallelogram law
2. Summing scalar components

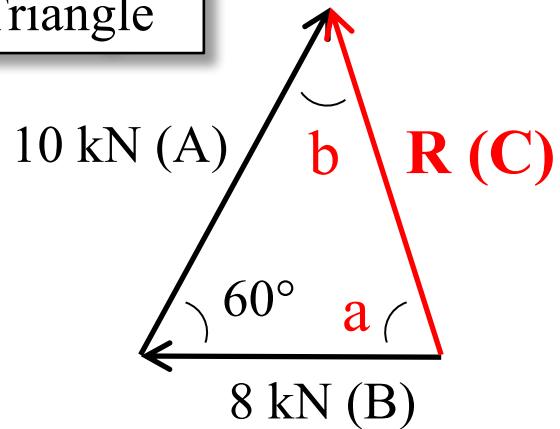


Parallelogram Law/Triangle Rule

Parallelogram Law



Force Triangle



Use the Law of Cosines to find the magnitude of the resultant

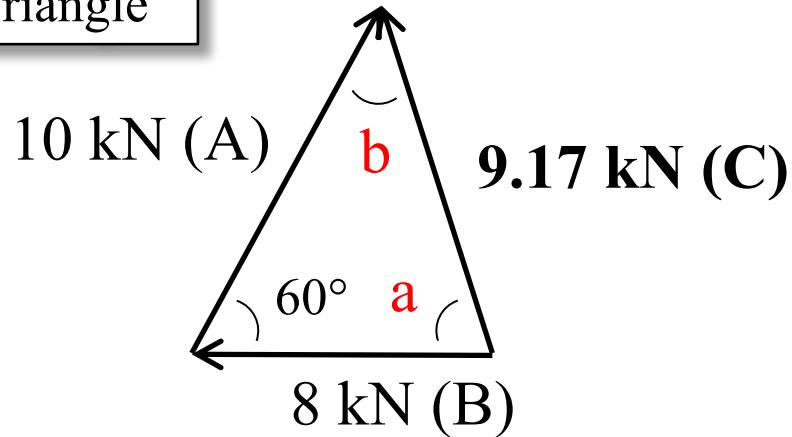
$$C^2 = A^2 + B^2 - 2AB \cos c$$

$$R^2 = (10 \text{ kN})^2 + (8 \text{ kN})^2 - 2(10 \text{ kN})(8 \text{ kN}) \cos 60^\circ$$

$$R = 9.17 \text{ kN}$$

Parallelogram Law/Triangle Rule

Force Triangle



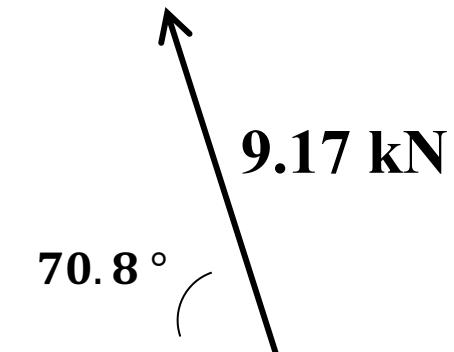
Use the Law of Sines to find the direction of the resultant

$$\frac{A}{\sin a} = \frac{C}{\sin c}$$

$$\frac{10 \text{ kN}}{\sin a} = \frac{9.17 \text{ kN}}{\sin 60^\circ}$$

$$\sin a = \frac{(10 \text{ kN})(\sin 60^\circ)}{9.17 \text{ kN}}$$

$$a = 70.8^\circ$$



Rectangular Force Components

Express 10 kN force in Cartesian Vector Form

$$F_{10x} = (10 \text{ kN}) \cos 60^\circ = 5 \text{ kN}$$

$$F_{10y} = (10 \text{ kN}) \cos 30^\circ = (10 \text{ kN}) \sin 60^\circ = 8.66 \text{ kN}$$

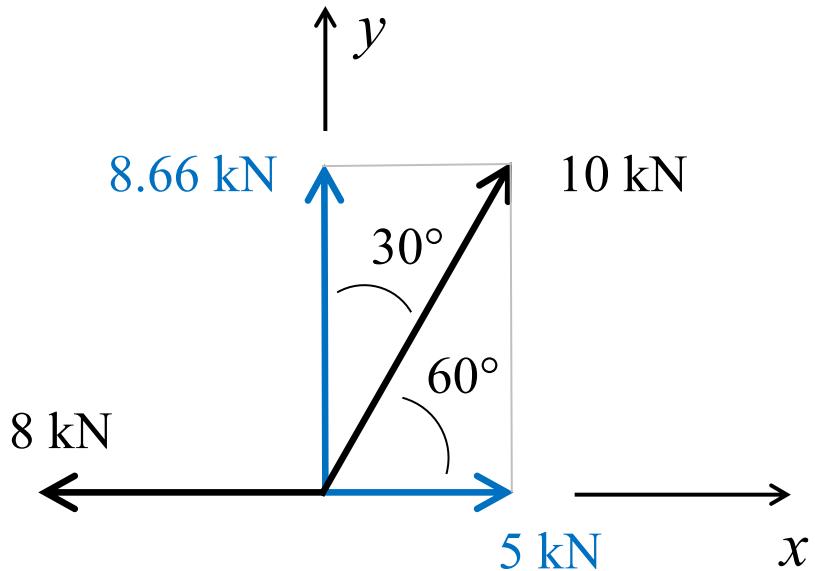
$$\mathbf{F}_{10} = 5\hat{i} + 8.66\hat{j} \text{ kN}$$

Express 8 kN force in
Cartesian Vector Form

$$\mathbf{F}_8 = -8\hat{i} \text{ kN}$$

Add scalar components

$$\mathbf{R} = \left(\sum F_x \right) \hat{i} + \left(\sum F_y \right) \hat{j}$$



Compare Results

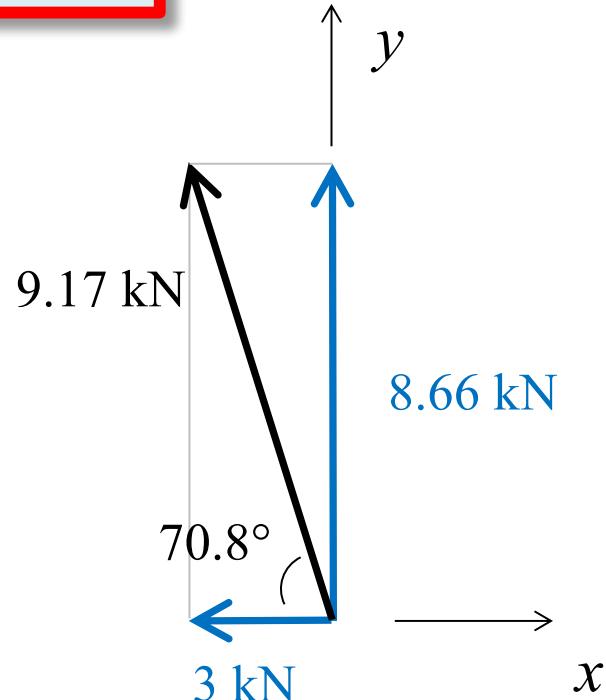
$$R = -3\hat{i} + 8.66\hat{j} \text{ kN}$$

Check magnitude and direction with previous result

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$R = \sqrt{(-3)^2 + (8.66)^2} = 9.17 \text{ kN} \quad \text{OK}$$

$$a = \tan^{-1} \left(\frac{8.66 \text{ kN}}{3 \text{ kN}} \right) = 70.8^\circ \quad \text{OK}$$



Note

In practice, using components is almost always the most efficient, particularly when the resultant of more than two forces is required.